

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

84[A, F].—M. LAL, *Expansion of $\sqrt{3}$ to 19600 Decimals*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, ms. of 2 typewritten pp. + computer printed table, deposited in UMT file.

The result here is very similar to Lal's previous work on $\sqrt{2}$. (See UMT 17, this volume of *Math. Comp.*, for a detailed review of that computation.) The method, computer, and computation time here are the same as in the previous computation. Each of 19 blocks of 1000 digits has a decimal-digit count and an evaluated χ^2 to 2D. The distribution appears to be random.

Lal's decimal-digit count for $\sqrt{3}$ at 14000D agrees with that of Takahashi & Sibuya (UMT 18, this volume). His digits 13901–14000 also were checked against theirs and complete agreement was found.

D. S.

85[F, JJ].—D. E. KNUTH & T. J. BUCKHOLTZ, *Tables of Tangent Numbers, Euler Numbers, and Bernoulli Numbers*, California Institute of Technology, Pasadena, California, January 1967, ms. of 311 computer sheets (unnumbered), 28 cm., deposited in the UMT file.

The first part of this manuscript consists of a 95-page table of the exact values of the first 404 Euler numbers, designated E_{2n} and taken here as all positive. The most extensive table of these numbers previously calculated appears to be that of Joffe [1], consisting of 50 entries, reproduced by Davis [2].

The remaining table in this manuscript is a 216-page compilation of the first 418 tangent numbers and corresponding numbers C_{2n} , for $n = 1(1)418$, from which the nonvanishing Bernoulli numbers can be obtained by the relation $B_{2n} = C_{2n} - \sum 1/p$, where the sum is taken over all primes p such that $(p - 1)|2n$, by virtue of the von Staudt–Clausen theorem. These primes are listed with each C_{2n} .

The tangent numbers, designated T_{2n-1} by the present authors, are integers defined by the Maclaurin expansion

$$\tan x = \sum_{n=1}^{\infty} T_{2n-1} x^{2n-1} / (2n - 1)!, \quad |x| < \frac{\pi}{2},$$

and are, accordingly, related to the Bernoulli numbers by the formula

$$(-1)^{n+1} T_{2n-1} = 2^{2n} (2^{2n} - 1) B_{2n} / (2n).$$

Previous tabulations of the exact values of T_{2n-1} and B_{2n} extend to at most $n = 30$ and $n = 110$, respectively [3], [4].

The calculations underlying the present tables constitute an extension of similar calculations of Bernoulli numbers carried out by Dr. Knuth in the course of his evaluation [5] of Euler's constant on a Burroughs 220 system. The present calculations, on the other hand, were performed on an IBM 7094 system, in a total running time of approximately 25 minutes. Further details of the method of calculation of these tables are set forth in a paper [6] appearing elsewhere in this journal.

J. W. W.

1. S. A. JOFFE, "Calculation of eighteen more, fifty in all, Eulerian numbers from central differences of zero," *Quart. J. Math.*, v. 48, 1920, pp. 193-271.
2. H. T. DAVIS, *Tables of the Mathematical Functions*, vol. II, revised edition, Principia Press of Trinity University, San Antonio, Texas, 1963.
3. J. PETERS, *Ten-Place Logarithm Tables*, new, revised edition, Ungar, New York, 1957, v. 1, *Appendix*, pp. 83-86, 88.
4. D. H. LEHMER, "An extension of the tables of Bernoulli numbers," *Duke Math. J.*, v. 2, 1936, pp. 460-464.
5. D. E. KNUTH, "Euler's constant to 1271 places," *Math. Comp.*, v. 16, 1962, pp. 275-281.
6. D. E. KNUTH & T. J. BUCKHOLTZ, "Computation of tangent, Euler, and Bernoulli numbers," *Math. Comp.*, v. 21, 1967, pp. 663-688.

86[G, H, I, M, X].—A. M. OSTROWSKI, *Solution of Equations and Systems of Equations*, Second Edition, Academic Press, New York, 1967, xiv + 338 pp., 24 cm. Price \$11.95.

In his inimitable style, the author produces a major revision of the first edition which is best described in the words of his Preface to the second edition:

"For this second edition, the entire text was thoroughly revised and much new material added so that the size of the book is almost doubled.

"The new material deals with those methods which can be used with the automatic computer without any special preparation. The Laguerre iteration and its modifications are extensively analyzed, since this iteration can be used, at least for polynomials with only real zeros, starting with an arbitrary real value. In two chapters and one appendix we treat the approximation of a zero by zeros of interpolating polynomials, as the extensive experimentations by D. I. Muller make it appear probable that in many cases this method is not sensitive with respect to the choice of the starting value. In several chapters we deal with the method of steepest descent. Although, in the case of one variable, the complete working through of this method to its practical use with a computer was achieved too late to be included in the book, the method of steepest descent gives a nonsensitive although rather slow approach for large classes of systems of equations, a subject that was somewhat neglected in the first edition. In this respect the four chapters preceding the generalization of the Newton-Raphson method to the case of several variables may be welcome to 'pure' as well as to 'applied' numerical analysts. Finally, the discussion of the theory of divided differences, added in this edition, may help to make this theory more widely known."

The review of the first edition by H. Schwerdtfeger [1] states in its final paragraph,

"This is certainly a remarkable book which will be welcome to everybody who wants suggestions for original work or for the preparation of lectures in numerical analysis with stress on mathematical rigour. The absence of exercises finds its counterbalance in the style, which calls for constant attention and collaboration of the reader. Finally, the excellent printing and generally pleasing appearance of the book deserve to be mentioned."

The effort required of the reader of the second edition is somewhat greater than was required for reading the first edition. The reward is correspondingly larger.

E. I.

1. Review B571, *Math. Reviews*, v. 23, 1962, p. 97.